Instabilities of a barotropic shear layer in a rotating fluid: asymmetries with respect to sgn(Ro)

ANA AGUIAR* and PETER READ

AOPP, Clarendon Lab., University of Oxford, UK

(Manuscript received October 1, 2005; in revised form April 26, 2006; accepted April 28, 2006)

Abstract
The aim of the work reported in this article is to provide new insights into the dynamics and instabilities of a barotropic shear layer in a rotating fluid. Above a critical value of horizontal stress, the flow within a bounded system in rotation is driven to an unstable limit, beyond which it develops chains of vortices. The number of these vortices depends not only upon the value of the stress imposed but also on the sense of the shear in some cases, highlighting discrepancies between earlier experiments. Quasi-geostrophic theory, however, predicts that there should be no qualitative differences with respect to the sign of the stress. We present laboratory experiments in cylindrical geometry, where a detached shear layer occurs tangential to the differentially rotating sections at the top and bottom of the tank. For stepped end walls, we found that the spatial organization of the flow patterns is a function of the sign of Rossby number. Furthermore, we observe a weak dependence of the azimuthal wavenumber on the sign of Rossby number.

1 Introduction

The Earth’s core and the eye of a hurricane are two examples of rotating fluid systems, where differential rotation results in a detached shear layer. A rapidly rotating homogeneous fluid in a cylindrical tank, the interior flow is typically characterized by a geostrophic equilibrium with weak inertia and viscous effects. However, near the solid end walls, a no-slip boundary condition leads to the existence of a thin horizontal layer of fluid where viscous effects become relevant and are balanced by Coriolis acceleration – an Ekman layer. Another type of internal boundary layer maybe associated with sharp changes in azimuthal velocity in the fluid’s interior – a detached vertical shear layer, known as a Stewartson layer. It allows the geostrophic interior to match the vertical boundary conditions, and most importantly returns any vertical mass flux which may be generated by the Ekman pumping. There are in fact two Stewartson layers with thicknesses proportional to different powers of the Ekman number: an inner one \( \propto E^{1/3} \) and an outer one \( \propto E^{1/4} \), Stewartson (1957).

The present study focuses on instabilities of a Stewartson layer. In practical experiments, that layer is induced by the differential rotation of concentric disks on the boundaries of a cylindrical container. For such conditions, the flow can be generally characterized by two parameters, namely, one that gives a measure of the strength of the differential rotation, compared to the system’s rotation rate – the Rossby number (Ro), and another one that is a measure of the viscous dissipation compared to the Coriolis acceleration (overall rotation rate) – the Ekman number (E),

\[
\begin{align*}
\text{Ro} & \equiv \frac{Ro}{2\tilde{\Omega}H} \quad E \equiv \frac{\nu}{\tilde{\Omega}H^2}
\end{align*}
\]

\( \tilde{\Omega} = \Omega + \omega/2 \) is the mean rotation rate, containing the background (\( \Omega \)) and the concentric disks’ (\( \omega \)) rotation rates; \( \nu \) is the kinematic viscosity of the fluid; \( H \) and \( R \) are the fluid depth and the radius of the inner disks, respectively (see Fig. 1).

HIDE and TITMAN (1967) [hereafter referred to as HT] did the first detailed experimental study of these