Equatorial global angular momentum: wind and mass terms

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Abstract

The normal mode theory of barotropic spherical flow predicts a close relation of the mass and wind terms of the equatorial components of global atmospheric angular momentum. Observations do not support this theoretical prediction. Here, an attempt is made to explain this discrepancy. Data are used to calculate modified equatorial wind terms where density is only a function of height. It is demonstrated that these modified wind terms capture only a small fraction of the true wind terms. This shows that covariances of density and wind as excluded in the normal mode theory contribute strongly to the wind terms. On the other hand, the results suggest that normal mode theory is useful in describing the response of equatorial angular momentum to mountains.

1 Introduction

The global angular momentum

\[ M = \int_V \rho r \times (v + \Omega \times r) dV \]  \hspace{1cm} (1.1)

(\(\rho\) density; \(r\) position; \(\Omega\) earth’s rotation; \(V\) volume of the atmosphere) of the atmosphere has with

\[ M = M_{i_1} + M_{i_2} + M_{j_1} + M_{j_2} \]  \hspace{1cm} (1.2)

a polar component \(M_i\) and two equatorial components \(M_e, M_y\). By definition the unit vector \(\mathbf{k}(i)\) points from the center of the earth towards the point \(\lambda = 0(\lambda = \pi/2)\), \(\varphi = 0\) on the surface of the earth (\(\lambda\) longitude, \(\varphi\) latitude). All three components have been evaluated and analyzed intensively (e.g. PEIXOTO and OORT, 1992; BELL, 1994; EGGER and HOINKA, 2002). The polar component is interesting because its changes are related to phenomena like the Madden-Julian oscillation and to variations of the length of the day. The equatorial components interact with the earth mainly through torques exerted by the equatorial bulge which lead in turn to polar motion.

It is customary to split the equatorial components into mass terms

\[ M_{\text{mass},x,y} = g^{-1} \int_S p_s \rho_a^2 \cos \varphi \sin \varphi \left( \frac{\cos \lambda}{\sin \lambda} \right) dS \]  \hspace{1cm} (1.3)

and wind terms

\[ M_{\text{wind},x,y} = \int_V \rho \left( -ua \sin \varphi \left( \frac{\cos \lambda}{\sin \lambda} \right) + va \left( \frac{\sin \lambda}{-\cos \lambda} \right) \right) dV \]  \hspace{1cm} (1.4)

(\(p_s\) surface pressure; \(u, v\) zonal, meridional wind components; \(S\) surface of the earth; \(dS = \rho_a \sin^2 \varphi d\varphi\)) where the earth’s radius \(a\) replaces \(|r|\) in (1.1) and where the upper (lower) harmonic function in the brackets relates to \(M_\varphi(M_\lambda)\).

It is clear from (1.3) that the surface pressure pattern must be of the form

\[ \rho_s \sim \cos \varphi \sin \varphi \exp(i\lambda) \]  \hspace{1cm} (1.5)

to contribute to the mass term. The quasigeostrophic barotropic vorticity equation suggests in turn that the atmospheric normal modes with streamfunction

\[ \psi = P_n^l(\varphi) \exp(i\lambda - i\sigma t) \]  \hspace{1cm} (1.6)

\((n = 1, 3; \sigma \text{ frequency})\) contribute most to the mass term (e.g. BELL, 1994) so that (1.6) should also provide estimates of the wind terms. BELL (1994) derived a simple

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